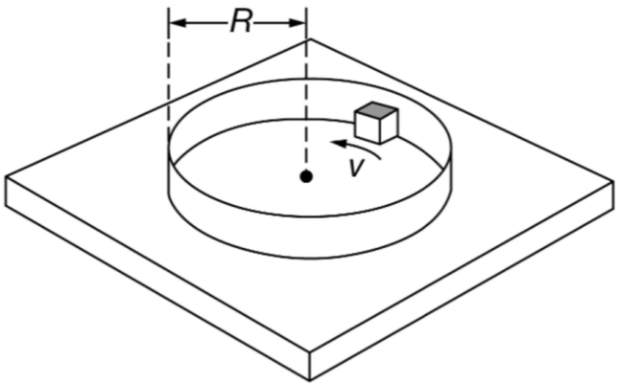
Dynamics 动力学

Newton’s law of motion 牛顿定律：

初等数学	高等数学
$\sum \vec{F} = m\vec{a} = m \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$	$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$

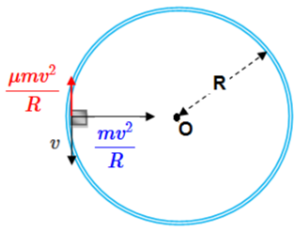
Example: A small block of mass m slide on a horizontal surface as it travels around the inside of a hoop of radius R. The coefficient of friction between the block and the wall is μ; therefore, the speed v of the block decreases. In terms of m, R, μ, v, find expressions for each of the following.



(a) The friction force on the block

Centripetal force $F_c = m\frac{v^2}{R}$

Frictional force $f = \mu F_c = \mu m\frac{v^2}{R}$



(b) The block’s tangential acceleration $a_t = \frac{dv}{dt}$

Tangential force $F_t = -f = -\mu m\frac{v^2}{R} = ma_t \Rightarrow a_t = \frac{dv}{dt} = -\mu\frac{v^2}{R}$

(c) The time required to reduce the speed of the block from an initial value v_0 to $v_0/2$

$\frac{dv}{dt} = -\mu\frac{v^2}{R} \Rightarrow \frac{dv}{v^2} = \frac{-\mu}{R}dt$

As time changes from 0 to t, velocity v changes from v_0 to $v_0/2$.

$\int_{v_0}^{v_0/2}\frac{dv}{v^2} = \int_0^t \frac{-\mu}{R}dt \Rightarrow \left.\frac{v^{-1}}{-1}\right|_{v_0}^{v_0/2} = \left(\frac{-\mu}{R}t\right)_0^t \Rightarrow \mathrm{t} = \frac{\mathrm{R}}{\mu v_0}$

Exercise 9: A time-dependent force, $\sum \vec{F} = (8.0\vec{i} - 4.0t\vec{j})N$, where t is in seconds, is exerted on a 2.0kg object initially at rest.

(a) At what time will the object be moving with a speed of 10.0 m/s?

(b) What total displacement has the object traveled through at this time?

(c) How far is the object from its initial position when its speed is 10.0m/s?

Exercise 10: A small block of mass m slides on a frictionless horizontal table. It is constrained to move inside a ring of radius R which is fixed to the table. At $t=0$, the block has a tangential velocity v_0 . The coefficient of friction between the block and the ring is μ. The velocity of the block at time t is?

Exercise 9: A time-dependent force, $\sum \vec{F} = (8.0\vec{\imath} - 4.0t\vec{\jmath})N$, where t is in seconds, is exerted on a 2.0kg object initially at rest.

(d) At what time will the object be moving with a speed of 10.0 m/s?

$$\sum \vec{F} = m\vec{a} \Rightarrow \vec{a} = \frac{\sum \vec{F}}{m} = \frac{8.0\vec{\imath} - 4.0t\vec{\jmath}}{2.0} = 4.0\vec{\imath} - 2.0t\vec{\jmath} = \frac{d\vec{v}}{dt}$$

$$\int_0^v d\vec{v} = \int_0^t (4.0\vec{\mathrm{i}} - 2.0t\vec{\mathrm{j}})dt \Rightarrow \vec{v} - 0 = 4.0t\vec{\mathrm{i}} - t^2\vec{\mathrm{j}}$$

$$|\vec{v}| = \sqrt{(4t)^2 + t^4} = 10.0 \Rightarrow t = 2.19\ (s)$$

(e) What total displacement has the object traveled through at this time?

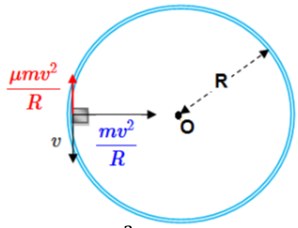
$$\int_0^{\Delta r} d\vec{r} = \int_0^t \vec{v}dt = \int_0^{2.19} (4.0t\vec{\mathrm{i}} - t^2\vec{\mathrm{j}})dt$$

$$\Rightarrow \Delta\vec{r} = \left(2.0t^2\vec{\mathrm{i}} - \frac{t^3}{3}\vec{\mathrm{j}}\right)_0^{2.19} = 9.61\vec{\mathrm{i}} - 3.51\vec{\mathrm{j}}\ \ (m)$$

(f) How far is the object from its initial position when its speed is 10.0m/s?

$$|\Delta\vec{r}| = \sqrt{9.61^2 + 3.51^2} = 10.2\ (m)$$

Exercise 10: A small block of mass m slides on a frictionless horizontal table. It is constrained to move inside a ring of radius R which is fixed to the table. At $t=0$, the block has a tangential velocity v_0 . The coefficient of friction between the block and the ring is μ. The velocity of the block at time is?



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As time changes from 0 to t, velocity v changes from v_0 to v.

$$\int_{v_0}^{v} \frac{dv}{v^2} = \int_0^t \frac{-\mu}{R}dt \Rightarrow \left.\frac{v^{-1}}{-1}\right|_{v_0}^{v} = \left(\frac{-\mu}{R}t\right)_0^t$$

$$\Rightarrow -\frac{1}{\mathrm{v}} + \frac{1}{\mathrm{v}_0} = \frac{-\mu}{R}t \Rightarrow \frac{\mathrm{v}_0 R}{R + \mathrm{v}_0\mu t}$$